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Torque Shaping Using Trigonometric Series Expansion for Slewing of Flexible Structures

Jinyoung Suk,* Jong-Yun Moon,* and Youdan Kim†
Seoul National University, Seoul 151-742, Republic of Korea

Optimal torque shaping based on a trigonometric series expansion of modified bang-bang input is obtained for simultaneous slewing and vibration suppression of flexible structures using a reaction wheel. Vibration energy is analyzed vis-a-vis maneuver time, torque shaping parameter, and number of terms to be used in a trigonometric series. Analytic expressions of performance indices and their derivatives are derived in modal space. A gain set of a tracking controller adopting Lyapunov stability theory is optimized to guarantee accurate targeting for the systems with/without external disturbances. Open-loop and closed-loop controls are applied to the flexible single-axis structure with a central hub and four cantilevered appendages. It is verified through numerical simulation that the flexible structure shows good slewing capability and excellent vibration suppression when adopting open-loop torque shaping. Moreover, the Lyapunov tracking control law shows no performance degradation during/after the maneuver even in the presence of external disturbance.

Introduction

RAPID retargeting and tracking missions will be needed for future applications where it is necessary to achieve precision pointing and vibration control of a structure during maneuvers. Therefore, simultaneous control of slewing and vibration suppression of flexible space structures has been an important research area during recent decades.[1–3] To solve this problem, a near-minimum-time tracking law based on Lyapunov stability theory is introduced,[2,3] and various torque shaping methods have been developed to avoid exciting the high-frequency dynamics of a flexible system.[4–8] Following the work of Junkins et al.,[2] we have sought an alternative to get the shaped torque that may not excite the natural frequency of the flexible appendages. This can also be effectively studied in the frequency region, and there are some interesting articles on the subject; Baruh and Tadikonda's[4] and Aspinwall's[5] are among them.

The objective of this paper is to present a control law modifying input torque shaping for slewing and vibration suppression of a flexible structure based on Fourier series expansion. We consider single-axis maneuvers of a rigid hub with four identical cantilevered flexible appendages. A mathematical model is developed using the finite element method assuming the flexible appendage to be an Euler–Bernoulli beam. A reaction wheel mounted on the central hub is used for slewing and vibration suppression. The input torque profile for slewing is represented by Fourier series expansion, and the coefficients of the trigonometric series are adopted as design variables to determine the input shape by an optimization procedure. Simple Fourier series expansion of the bang-bang type command is taken as the reference torque, and by minimizing the reference torque and modified-shaping torque, the peak magnitude of the resultant torque may be lowered. The performance index consists of three objective functions: 1) vibration energy of the appendage during the maneuver, 2) vibration energy at the end of the maneuver, and 3) mean square error between the reference torque and the modified shaping torque. Constraints are imposed on final target angle to guarantee rest-to-rest maneuver. The analytic expression of the cost functions is derived using the modal coordinates. To apply a gradient-based optimization technique, derivatives of each objective function and constraint with respect to the design parameters are required. To obtain accurate and efficient results, a sensitivity analysis that analytically determines the derivatives of the performance index and constraint is also performed.

This paper is organized as follows. First, the hub–appendage structure with the reaction wheel is modeled to derive the governing equation. The vibration energy of the structure with the shaped control input based on Fourier series expansion is investigated with respect to the shaping parameter of the reference torque, maneuver time, and the number of trigonometric functions included in the series. Analytic expressions of the objective functions are derived for open-loop control with the reaction wheel using the modal coordinates. The sensitivity analysis is also performed for the performance index and constraint with respect to the design parameters. An optimal tracking controller is designed using Lyapunov's stability theory to get the feedback type control law. The control gain set is optimized so that the overall system may follow the reference maneuver in the presence of external disturbance. Finally, numerical results are shown and discussed.

Optimal Torque Shaping

Dynamic characteristics of three-axes stabilization flexible structures can be described using structural modes in addition to rigid modes: Three translational modes are used to express the orbital dynamics and three rotational modes determine the orientation of structures. In this study, a single-axis maneuver with vibration modes is considered to investigate the dynamics of the flexible structures and to design the optimal input torque.

Consider the planar rotational/vibrational dynamics of a flexible structure consisting of a rigid hub with four identical cantilevered flexible appendages, as shown in Fig. 1. Considering rigid and antisymmetric structural modes, the governing equation of motion is obtained using the finite element method[9]

$$M\ddot{\mathbf{x}}(t) + K\mathbf{x}(t) = Fu(t) \tag{1}$$

where

$$M = \begin{bmatrix} J_h + 4M_{\theta\theta} & 4M_{\theta v} \\ 4M_{\theta v}^T & 4M_{vv} \end{bmatrix}, \qquad K = \begin{bmatrix} 0 & \mathbf{0}_{1\times 2n}^T \\ \mathbf{0}_{2n\times 1} & 4K_{vv} \end{bmatrix}$$

$$F = \begin{bmatrix} 1 \\ 0 \\ \cdots \\ 0 \end{bmatrix}$$

Note that $M_{\theta\theta}$ is a moment of inertia of each appendage, $M_{\theta v}$ is a coupling matrix between the rigid and structural modes, and M_{vv} and K_{vv} are mass and stiffness matrices of the beam mode, respectively.[9]

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*Graduate Student, Department of Aerospace Engineering.
†Associate Professor, Department of Aerospace Engineering. E-mail: ydkim@plaza.snu.ac.kr. Senior Member AIAA.

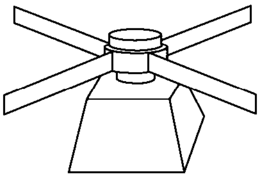


Fig. 1 Hub–appendage flexible structure system.

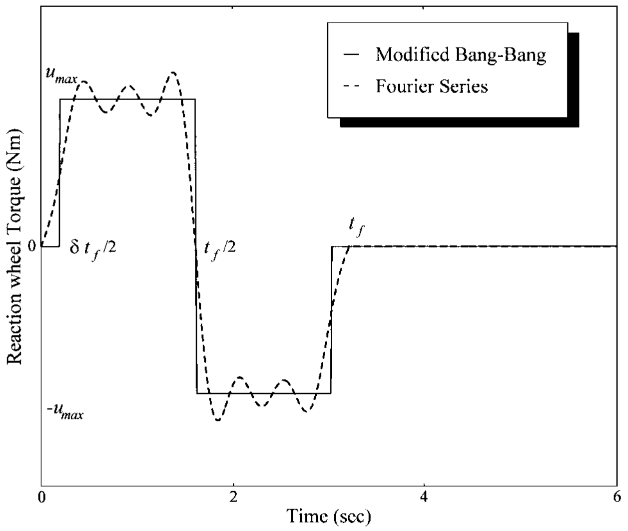


Fig. 2 Modified bang-bang input and Fourier series expansion.

The states are composed of the rigid mode and the flexible dynamics of the elastic modes as follows:

$$x^T = [\Theta \quad w_1 \quad \theta_1 \quad w_2 \quad \theta_2 \quad \cdots \quad w_n \quad \theta_n]$$

where Θ, w_i, and θ_i are hub angle, bending displacement, and angle of ith finite element node, respectively. It is well known that the constrained minimum-time solution requires a bang-bang type control input. However, an abrupt change of input torque excites the structural modes, which degrades control performance and causes residual vibration after the maneuver. In this study, Fourier series expansion is used to reshape bang-bang input so that structural modes may not be excited much, and modified bang-bang input is considered to shape the Fourier series using the shaping parameter δ, as shown in Fig. 2. Introduction of δ enables much reduction of residual vibration. The Fourier series of the modified bang-bang input can be expressed as follows:

$$u(t) = \sum_{i=1}^{N} a_i \sin\left\{\frac{i\pi t}{t_f/2}\right\} \tag{2}$$

where

$$a_i = -2(u_{\max}/i\pi)\cos i\pi\{1 - \cos(1-\delta)i\pi\}$$

and $u_{\max}$ is the maximum value of the modified bang-bang input as a function of target angle Θ_f, which is obtained as follows:

$$u_{\max} = \frac{4(J_h + 4M_{\theta\theta})\Theta_f}{(1-\delta)^2 t_f^2}$$

In this study, target final angle is set to be 60 deg. The maneuver time t_f, the number of terms N of the trigonometric series, and the shaping parameter δ control the smoothness of the modified shaping torque and, therefore, the vibration energy of the system. A judicious set of parameters (t_f, N, and δ) for the global minimum energy may be obtained by roughly searching the whole region on the three parameters. In this study, the first seven terms are included to shape the control input. Figure 3 shows the vibration energy of flexible structures with respect to δ and t_f, whereas N is set to 7. The vibration energy is defined as a weighted sum of kinetic and potential energy of the structure during and at the end of maneuver as follows:

Table 1 Vibration energy and maximum torque characteristics for specified maneuver time (δ = 0.12)

Maneuver time, s:	2.89	3.22	3.54	3.86	4.18
Vibration energy	461	202	100	64	48
Maximum torque, Nm	1.14	0.92	0.76	0.64	0.55

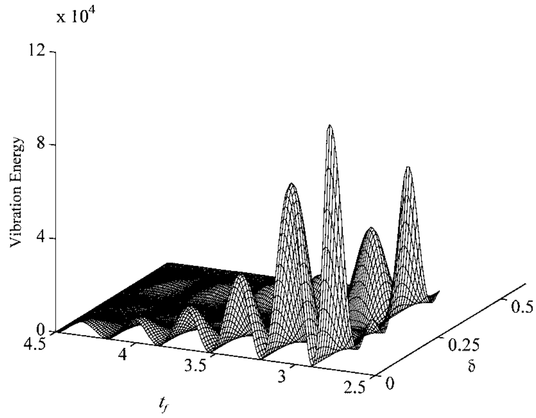


Fig. 3 Vibration energy for various parameters.

$$E = \gamma_1 \int_0^{t_f} \left\{4x_v^T K_{vv} x_v + 4\dot{x}_v^T M_{vv} \dot{x}_v\right\} \mathrm{d}t + \gamma_2 \left[4x_v^T K_{vv} x_v + 4\dot{x}_v^T M_{vv} \dot{x}_v\right]\Big|_{t=t_f}$$

where γ_i is a weighting factor and x_v is the coordinate vector, which consists of the transverse deflection and rotation at each node of the appendage. The weighting factor γ_i is chosen to balance the contribution of each term.

Figure 3 shows that the vibration energy reaches local maxima whenever the maneuver time approaches the multiple of the frequency of the first structural mode. Viewing the vibration energy surface in the δ axis, we can find a valley extracting $\delta = 0.12$. Also, viewing in the t_f axis, we can find that the vibration energy decreases as t_f increases, showing local minima and maxima. It is obvious that the initial setup for t_f should be one of those points that render local minima. However, the maximum torque gets higher when t_f becomes smaller, as listed in Table 1. In this study, t_f is set to be 3.22 s, considering local minimum of the vibration energy and maximum allowable torque for the reaction wheel.

To perform the modal coordinate transformation, we introduce the general modal coordinate transformation

$$x = \Phi\eta \tag{3}$$

where Φ is the modal matrix satisfying the biorthogonality conditions and

$$\Phi^T M \Phi = I, \qquad \Phi^T K \Phi = \mathrm{diag}\left\{\ldots, \omega_i^2, \ldots\right\} \equiv \Lambda \tag{4}$$

For the rest-to-rest maneuver, the transformed equation of motion with zero initial conditions becomes

$$\ddot{\eta}_i + \omega_i^2 \eta_i = \psi_i \sum_{j=1}^{N} a_j \sin\omega_j t \tag{5}$$

where ψ_i is the ith element of $\Phi^T F$ and ω_j is $2\pi j/t_f$. For the case that $\omega_i \neq \omega_j$, the solution of Eq. (5) is given by

$$\eta_i(t) = A_i \sin\omega_i t + B_i \cos\omega_i t + \sum_{j=1}^{N} c_{ij} \sin\omega_j t \tag{6}$$

The coefficient c_{ij} can be chosen as

$$c_{ij} = \frac{\psi_i a_j}{\omega_i^2 - \omega_j^2} \tag{7}$$

The other coefficients A_i and B_i can also be obtained using initial conditions as follows:

$$A_i = -\sum_{j=1}^{N} \frac{\omega_j \psi_i a_j}{\omega_i\left(\omega_i^2 - \omega_j^2\right)}, \qquad B_i = 0 \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (6) yields the solution in modal coordinates as

$$\eta_i(t) = \sum_{j=1}^{N} \alpha_{ij}\left(\sin\omega_j t - \frac{\omega_j}{\omega_i}\sin\omega_i t\right) \tag{9}$$

$$\dot\eta_i(t) = \sum_{j=1}^{N} \beta_{ij}\{\cos\omega_j t - \cos\omega_i t\} \tag{10}$$

where

$$\alpha_{ij} = \frac{\psi_i a_j}{\omega_i^2 - \omega_j^2}, \qquad \beta_{ij} = \frac{\omega_j \psi_i a_j}{\omega_i^2 - \omega_j^2}$$

In this study, performance index is defined so that the vibration energy during the maneuver and the residual variation are minimized. The performance index consists of three objective functions: 1) vibration energy of the appendage during the maneuver, 2) vibration energy at the end of the maneuver, and 3) the mean square error between the modified bang-bang input and the reaction wheel torque to be optimized. The performance index can be expressed as follows:

$$J = \sum_{i=1}^{3} \beta_i J_i$$

where β_i denotes a weighting factor.

The objective function J_1 represents the vibration energy of the flexible structure during the maneuver as follows:

$$J_1 = \int_0^{t_f} \sum_{i=0}^{m} \left(\omega_i^2\eta_i^2 + \dot\eta_i^2\right) \mathrm{d}t \tag{11}$$

where η_0 is the modal coordinate of the rigid mode, η_i $(i = 1, 2, \ldots, m)$ is the modal coordinate of the structural modes, and m is the number of flexible modes to be considered. Using Eqs. (9) and (10) in Eq. (11), the analytic expression of J_1 can be obtained as

$$\int_0^{t_f} \omega_i^2\eta_i^2(t)\,\mathrm{d}t = \sum_{j=1}^{N}\omega_i^2\alpha_{ij}^2\frac{t_f}{2} + \sum_{j=1}^{N}\sum_{k=1}^{N}\omega_i^2\alpha_{ij}\alpha_{ik} \times \left[\frac{\omega_k}{2\omega_i}(P_{ij} - M_{ij}) + \frac{\omega_j}{2\omega_i}(P_{ik} - M_{ik}) + \frac{\omega_j\omega_k}{2\omega_i^2}\left(t_f - \frac{\sin 2\omega_i t_f}{2\omega_i}\right)\right] \tag{12}$$

$$\int_0^{t_f} \dot\eta_i^2(t)\,\mathrm{d}t = \sum_{j=1}^{N}\beta_{ij}^2\frac{t_f}{2} + \sum_{j=1}^{N}\sum_{k=1}^{N}\beta_{ij}\beta_{ik}\left[\frac{t_f}{2} + \frac{\sin 2\omega_i t_f}{4\omega_i} - \frac{1}{2}(P_{ij} + M_{ij} + P_{ik} + M_{ik})\right] \tag{13}$$

where

$$P_{ij} = \frac{\sin(\omega_i + \omega_j)t_f}{\omega_i + \omega_j}, \qquad M_{ij} = \frac{\sin(\omega_i - \omega_j)t_f}{\omega_i - \omega_j}$$

$$P_{ik} = \frac{\sin(\omega_i + \omega_k)t_f}{\omega_i + \omega_k}, \qquad M_{ik} = \frac{\sin(\omega_i - \omega_k)t_f}{\omega_i - \omega_k}$$

Note that the frequency of the rigid mode, ω_0, is zero.

The objective function J_2 represents the residual vibration energy at the end of the maneuver

$$J_2 = \sum_{i=0}^{m}\left(\omega_i^2\eta_i^2 + \dot\eta_i^2\right)\Big|_{t_f} \tag{14}$$

Substituting Eqs. (9) and (10) in Eq. (14), we can obtain the expression of the residual vibration energy at the final time as follows:

$$\omega_i^2\eta_i^2(t_f) = \sum_{j=1}^{N}\sum_{k=1}^{N}\alpha_{ij}\alpha_{ik}\omega_j\omega_k\sin^2\omega_i t_f \tag{15}$$

$$\dot\eta_i^2(t_f) = \sum_{j=1}^{N}\sum_{k=1}^{N}\beta_{ij}\beta_{ik}(1 - \cos\omega_i t_f)^2 \tag{16}$$

To reduce peak magnitude, we adopt the objective function J_3 that takes the mean square error between the modified bang-bang input (shown in Fig. 2) and the reaction wheel torque to be optimized. It can be formulated as follows:

$$J_3 = \left\{\frac{u_{\max}^2(1-\delta)t_f}{2} + \frac{t_f}{4}\sum_{i=1}^{N}a_i^2 + \frac{u_{\max}t_f}{\pi}\sum_{i=1}^{N}\frac{a_i}{i}[\cos\pi i - \cos\delta\pi i]\right\} \tag{17}$$

Two constraints are imposed on final conditions to guarantee the rest-to-rest maneuver. Final target angle should be equal to Θ_f, and the angular velocity should be zero at the end of maneuver. Consider the rigid-body equation acted on by the control input

$$I\ddot\Theta(t) = \sum_{i=1}^{N} a_i \sin\omega_i t \tag{18}$$

where $I = J_h + 4M_{\theta\theta}$, the total moment of inertia. Integrating Eq. (18) using the initial rest conditions and evaluating the value at $t = t_f$ yield

$$I\dot\Theta(t_f) = \sum_{i=1}^{N}\frac{a_i}{\omega_i}(1 - \cos\omega_i t_f) = 0 \tag{19}$$

$$I\Theta(t_f) = \sum_{i=1}^{N}\frac{a_i}{2\pi i}t_f^2 \tag{20}$$

Equation (19) is satisfied automatically because $\omega_i = 2\pi i/t_f$. Therefore, the minimum requirement to achieve the rest-to-rest maneuver can be expressed by Eq. (20). Derivatives of objective functions are required for applying the gradient-based optimization algorithm. The analytic expression of the performance index during the optimization process not only reduces computational time, but also gives a better solution for the optimization. Detailed analytic expressions of derivatives are given in the Appendix.

Lyapunov Tracking Control Law

In the preceding section, the analytic formula for evaluating objective functions for optimal open-loop torque input imposed on the reaction wheel is derived. The control law using optimized shaped torque input usually yields good performance for the system without external disturbance. However, a closed-loop control law should be included to guarantee the performance of slewing and vibration suppression in the presence of significant external disturbance. In this section, the optimal tracking controller is designed based on the Lyapunov stability theorem to follow the reference maneuver, as well as to suppress inherent vibration caused by the motion of the central hub.

The reference maneuver of the flexible structure is designed as follows:

$$I\ddot\Theta_{\mathrm{ref}}(t) = u_{\mathrm{ref}}(t) \tag{21}$$

where $u_{\rm ref}$ is the open-loop optimized shaped torque for the reaction wheel derived in the preceding section. The following error energy Lyapunov function is selected to design the globally stable tracking controller considering tracking error energy and vibration energy of the whole system:

$$2U = a_1 J_h \delta\dot{\Theta}^2 + a_2 \delta\Theta^2 + 4a_3 \left\{ \int_{l_0}^{l} \rho[\delta\dot{w} + x\delta\dot{\Theta}]^2 \,\mathrm{d}x + \int_{l_0}^{l} EI(\delta w'')^2 \,\mathrm{d}x \right\} \tag{22}$$

Using Eq. (22), the Lyapunov control law can be obtained as follows[3]:

$$u(t) = u_{\rm ref}(t) - g_1(\Theta - \Theta_{\rm ref}) - g_2(\dot{\Theta} - \dot{\Theta}_{\rm ref}) - g_3 \int_{l_0}^{l} \{\rho x(\ddot{w} + x\ddot{\Theta}) - \rho x^2 \ddot{\Theta}_{\rm ref}\} \,\mathrm{d}x \tag{23}$$

where l_0 and l are the radius of hub and the length of appendage plus the radius of hub, respectively. Using Eq. (23) in connection with Eq. (1), the control input can be expressed as a function of the system states and their first/second derivative as follows:

$$u(t) = u_{\rm ext} - g_1\Theta - g_2\dot{\Theta} - g_3 \int_{l_0}^{l} \rho x^2 \,\mathrm{d}x \ddot{\Theta} - \frac{g_3}{4} M_{\theta v} \ddot{\boldsymbol{x}}_v \tag{24}$$

where

$$u_{\rm ext}(t) = g_3^* \ddot{\Theta}_{\rm ref} + g_2 \dot{\Theta}_{\rm ref} + g_1 \Theta_{\rm ref} \tag{25}$$

$$g_3^* = J_h + (4 + g_3) M_{\theta\theta} \tag{26}$$

$$\boldsymbol{x}_v = [w_1 \quad \theta_1 \quad \cdots \quad w_n \quad \theta_n]^T \tag{27}$$

and J_h is the rotary inertia of the hub. Substituting Eq. (24) for Eq. (1), the following closed-loop equation of motion is obtained:

$$\bar{M}\ddot{\boldsymbol{x}} + \bar{C}\dot{\boldsymbol{x}} + \bar{K}\boldsymbol{x} = F u_{\rm ext} \tag{28}$$

where

$$\bar{M} = M + g_3 F \left[\int_{l_0}^{l} \rho x^2 \,\mathrm{d}x \quad \frac{1}{4} M_{\theta v} \right] \tag{29}$$

$$\bar{C} = F[g_2 \quad \boldsymbol{0}_{1\times 2n}] \tag{30}$$

$$\bar{K} = K + F[g_1 \quad \boldsymbol{0}_{1\times 2n}] \tag{31}$$

In general, computational time and numerical accuracy are improved utilizing the coordinate transformation. Equation (28) can be made symmetric by multiplying from the second to the last row of $\bar{M}$, $\bar{C}$, and $\bar{K}$ by $(1 + g_3/4)$, and symmetry is guaranteed irrespective of the control gains. To optimize the control gain set and thereby maximize the performance of slewing and vibration suppression, the following performance index is defined for the closed-loop tracking control law[2]:

$$J(\boldsymbol{p}) = \kappa_1 \int_0^{t_f} (\Theta - \Theta_{\rm ref})^2 \,\mathrm{d}t + \kappa_2 \int_0^{t_f} (\dot{\Theta} - \dot{\Theta}_{\rm ref})^2 \,\mathrm{d}t + 4\kappa_3 \times \int_0^{t_f} \int_{l_0}^{l} \left\{ \kappa_4 \rho[\dot{\omega} + x(\dot{\Theta} - \dot{\Theta}_{\rm ref})]^2 + (EI\omega'')^2 \right\} \mathrm{d}x \,\mathrm{d}t \tag{32}$$

where parameter vector $\boldsymbol{p}$ consists of the control gain set g_i and κ_i are selected in consideration of the relative significance between the competitive objective functions. The choice of Eq. (32) as a performance measure reflects a desire to make the tracking error and vibration energy as small as possible.

Table 2 Configuration parameters of the flexible structure

Parameter	Value
Hub radius, m	0.20
Rotary inertia of hub, kg m^2	1.2732
Mass density of appendage, kg/m^3	2800
Young's modulus of appendage, N/m^2	7.5842×10^{10}
Thickness of appendage, m	0.0010
Width of appendage, m	0.0635
Length of appendage, m	0.8100

Numerical Simulation

A 60-deg rest-to-rest maneuver is simulated for the flexible structure shown in Fig. 1. Physical properties of the flexible structure are listed in Table 2. Flexible appendages are modeled using 10 finite elements. After investigating the vibration energy of the flexible structure as shown in Fig. 3, δ and t_f are set to 0.12 and 3.22, respectively. The following two steps are used to optimize parameters related to the torque shaping. First, modified bang-bang torque is designed, as shown in Fig. 2, and initial values are assigned for Fourier coefficients in Eq. (2). Second, input coefficients for the reaction wheel are obtained such that they minimize the performance index while satisfying the constraint Eq. (20). Optimized input shaping parameters for the reaction wheel are computed as follows:

$$a_1 = 1.0098, \qquad a_2 = 0.1504, \qquad a_3 = 0.1968$$

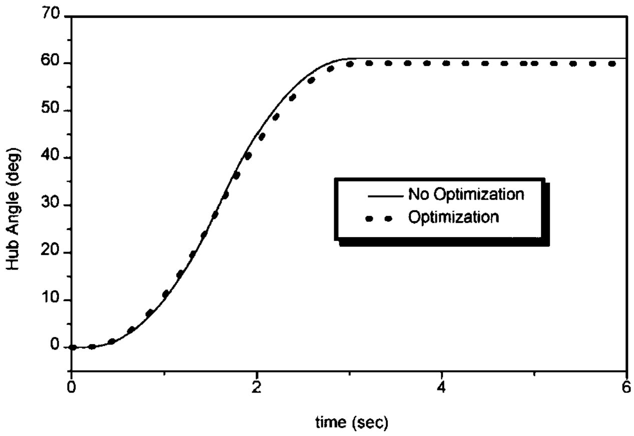
$$a_4 = -0.0364, \qquad a_5 = -0.0046$$

$$a_6 = -0.0905, \qquad a_7 = -0.0640$$

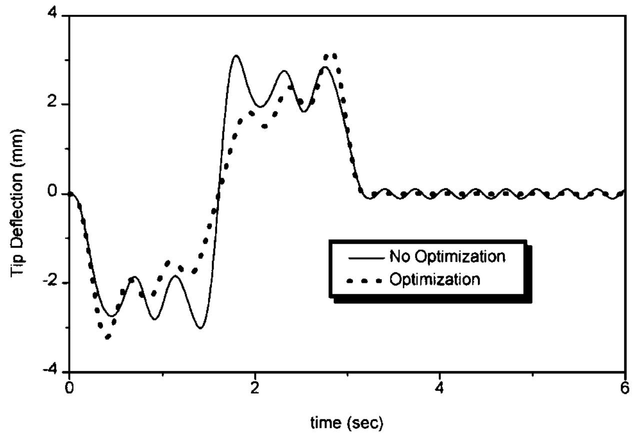
The maximum number of coefficients is limited to seven, because the results using more coefficients are almost similar to those using only the first seven coefficients.

Figures 4a–4c show simulated system responses of hub angle, tip deflection of the appendage, and control torque driven by the reaction wheel, respectively, using optimal open-loop torque shaping. As shown in Fig. 4a, optimal torque shaping of the reaction wheel results in good tracking performance, whereas torque shaping using only Fourier coefficients without optimization shows some discrepancy in final target angle. We can state that optimization of torque shaping parameters results in good vibration suppression performance while satisfying final target angle. Figure 4b shows enhanced performance of vibration suppression using efficient torque shaping by vanishing the residual vibration after the maneuver. Moreover, we can enjoy improved vibration control using the reaction wheel. Figure 4c shows comparative results on the reaction wheel torque. This gives some indication of how the control input is rendered to provide an efficient torquing for both slewing and vibration control. To achieve efficient vibration control of the appendage, elastic energy caused by the appendage should be absorbed in the actuator torquing; thereby, vibrating control torque should be entailed, whose frequency spectra corresponds to those of the natural frequencies of the whole flexible system. Note that the proposed open-loop torque shaping methodology deals with the deterministic case so that we assume all of the dynamic information on the structure to be known. Therefore, the slew performance, as well as the vibration suppression performance, might be sensitive to parameter variation. For example, the variation of EI does not affect the slew performance, but does the vibration suppression: As the appendage becomes stiffer, the vibration during the maneuver gets smaller and vice versa. The variation of mass property affects the slew performance as well. The primary objective of this work is to illustrate the proposed open-loop torque shaping methodology. The robustness problem of the methodology is still an open question for further research.

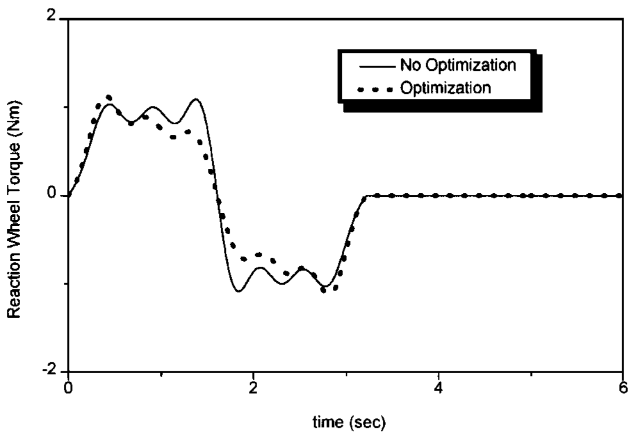
To deal with the system with external disturbances, the control gain set shown in Eq. (23) is optimized to follow the optimal reference profile applying the reaction wheel torque shown in Fig. 4. Weighting factors κ_i in the performance index are chosen as 10^7, 10^5, 70, and 35, respectively. Maximum and minimum values of the



a) Hub angle



b) Tip deflection



c) Reaction wheel torque

Fig. 4 Time histories using open-loop optimization.

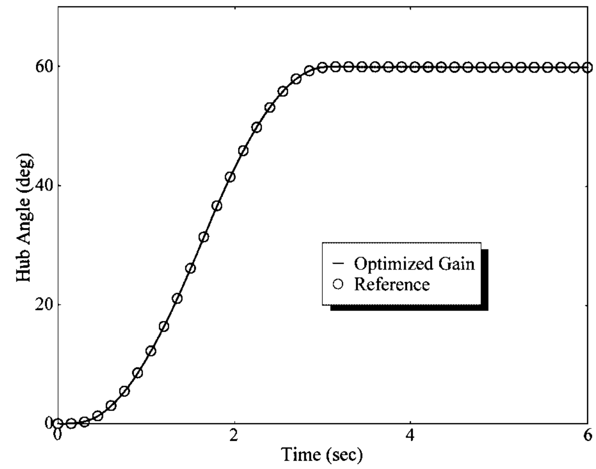


Fig. 5 Slewing performance, closed-loop hub angle response with external disturbance.

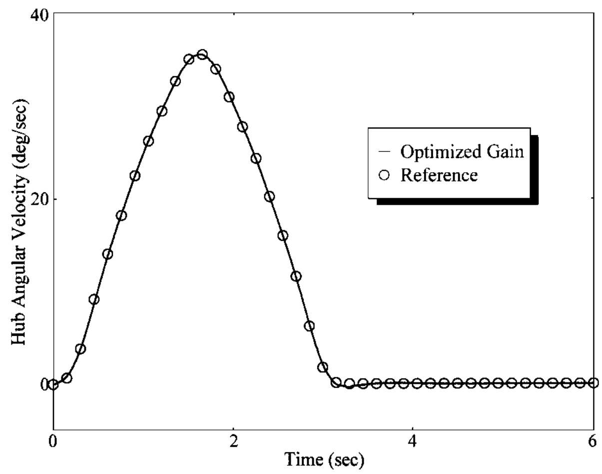


Fig. 6 Slewing performance, closed-loop hub angular velocity response with external disturbance.

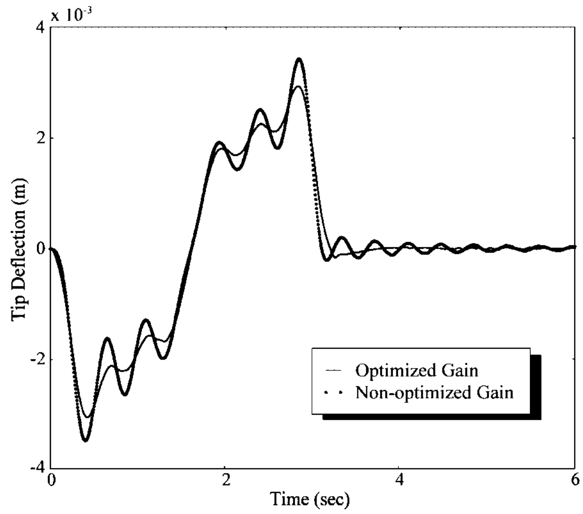


Fig. 7 Vibration suppression performance: closed-loop with external disturbance.

control gain set are assigned as follows so that 1) the actuators may not saturate and 2) stability is guaranteed:

$$0 \le g_1 \le 1000, \qquad 0 \le g_2 \le 1000, \qquad -3.99 \le g_3 \le 1000$$

The optimal control gain set is determined as follows for a 60-deg maneuver:

$$\boldsymbol{g} = [989.99 \quad 240.02 \quad 339.99]$$

The optimal gain set obtained shows no better results than those in open-loop control because there is no external disturbance torque, and we can see that open-loop optimal torque shaping provides the best results. However, we can see the distinguishable performance when adopting the closed-loop tracking controller in the presence of disturbance. Figures 5–8 display the comparative time histories of hub angle, hub angular velocity, tip deflection of the appendage, and control torque driven by the reaction wheel, with substantial quasirandom (0.05-Nm strength) disturbance torques acting on the reaction wheel. External disturbances are considered to be caused by certain nonlinear effects on the actuator dynamics that are highly correlated in time. It can be shown that the time histories of closed-loop control track the reference maneuver quite well in spite of the disturbing torque. All these results validate the superior performance of slewing and vibration suppression capability when adopting

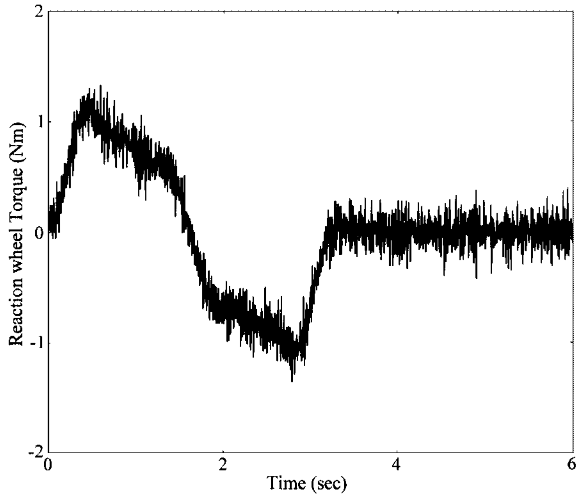


Fig. 8 Reaction wheel torque input: closed-loop with external disturbance.

closed-loop control based on Lyapunov's stability theory. Moreover, as shown in Fig. 7, simulation results through gain optimization show outstanding performance in vibration suppression aspects. As a consequence, we can conclude that the Lyapunov tracking control law guarantees the slewing performance of flexible spacecraft, and the performance in vibration control can be improved via an efficient gain optimization procedure.

Conclusion

An effective input torque shaping method is developed to implement both slewing and vibration control of flexible structures. Open-loop torque shaping is based on Fourier series expansion of modified bang-bang input to achieve nearly minimum time maneuver. Using open-loop optimization results, the tracking controller is designed to guarantee the global stability of the system in the sense of Lyapunov. Time histories adopting open-loop optimization show good performance on the tracking reference maneuver, matching final target angle and zeroing residual vibration after the maneuver. Moreover, it is verified that, using the tracking controller based on Lyapunov's stability theory, stable maneuver and vibration control is possible in the presence of substantial external disturbances. The proposed torque shaping method is easy to implement for experimental work because the resulting torque is continuous, based on trigonometric series expansion. The method developed will provide an efficient basis for implementing realizable torque shaping of slewing flexible structures.

Appendix: Derivatives of Objective Functions and Constraint Equation

The partial derivative of J_1 [Eq. (11)] with respect to a_k consists of the following two components:

$$\frac{\partial}{\partial a_k}\left\{\int_0^{t_f}\omega_i^2\eta_i^2(t)\,\mathrm{d}t\right\} = \frac{\psi_i^2\omega_i^2 t_f a_k}{\left(\omega_i^2-\omega_k^2\right)^2}$$

$$+\sum_{j=1}^{N}\frac{2\psi_i^2\omega_i^2 a_j}{\left(\omega_i^2-\omega_k^2\right)\left(\omega_i^2-\omega_j^2\right)}\left[\frac{\omega_k}{2\omega_i}(P_{ij}-M_{ij})\right.$$

$$\left.+\frac{\omega_j}{2\omega_i}(P_{ik}-M_{ik})+\frac{\omega_j\omega_k}{2\omega_i^2}\left\{t_f-\frac{\sin 2\omega_i t_f}{2\omega_i}\right\}\right]$$

$$\frac{\partial}{\partial a_k}\left\{\int_0^{t_f}\dot{\eta}_i^2(t)\,\mathrm{d}t\right\}$$

$$=\frac{\omega_k^2\psi_i^2 t_f a_k}{\left(\omega_i^2-\omega_k^2\right)^2}+\sum_{j=1}^{N}\frac{2\omega_j\omega_k\psi_i^2 a_j}{\left(\omega_i^2-\omega_k^2\right)\left(\omega_i^2-\omega_j^2\right)}$$

$$\times\left[-\frac{1}{2}(P_{ij}+M_{ij})-\frac{1}{2}(P_{ik}+M_{ik})+\frac{t_f}{2}+\frac{\sin 2\omega_i t_f}{4\omega_i}\right]$$

The partial derivative of J_2 [Eq. (15)] with respect to a_k consists of the following two components:

$$\frac{\partial}{\partial a_k}\left\{\omega_i^2\eta_i^2(t_f)\right\}=2\sum_{j=1}^{N}a_j\frac{\psi_i^2\omega_j\omega_k\sin^2\omega_i t_f}{\left(\omega_i^2-\omega_k^2\right)\left(\omega_i^2-\omega_j^2\right)}$$

$$\frac{\partial}{\partial a_k}\left\{\dot{\eta}_i^2(t_f)\right\}=2\sum_{j=1}^{N}a_j\frac{\psi_i^2\omega_j\omega_k(1-\cos\omega_i t_f)^2}{\left(\omega_i^2-\omega_k^2\right)\left(\omega_i^2-\omega_j^2\right)}$$

The partial derivative of J_3 [Eq. (18)] with respect to a_k is as follows:

$$\frac{\partial J_3}{\partial a_k}=\beta_3\left\{\frac{t_f}{2}a_k+\frac{u_{\max}t_f}{\pi k}(\cos\pi k-\cos\delta\pi k)\right\}$$

The constraint equation [Eq. (20)] and the partial derivative are as follows:

$$g=\sum_{i=1}^{N}\frac{a_i}{i}-\frac{2\pi I}{t_f^2}\Theta_f=0,\qquad \frac{\partial g}{\partial a_k}=\frac{1}{k}$$

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